S5 MATHS PAPER 1/2 (ALGEBRA AND CALCULUS)

To be done in two weeks (23/03/2020 To 4/04/2020)

Attempt all the questions. Where possible some **hints** have been given.

1. Find the value of x if $\log_x 10.24 = 2$.

2. Solve the equations (i) $\log(a^2 + 2a) = 0.9031$ (ii) $3.2^{2x} = 8.45$

3. Solve the equations $2^x + 4^y = 12$, $3(2^x) - 2(2^{2y}) = 16$.
   Hence show that $4^x + 4(3)^{2y} = 100$.

   Hint: Let $a = 2^x$, $b = 2^{2y}$, form two simultaneous equations and solve them.

4. Solve the simultaneous equations $2^{x+y} = 6^y$, $3^x = 6(2^y)$.

5. Given that $p = \log_a(bc)$, $q = \log_b(ca)$ and $r = \log_c(ab)$, deduce that $p + q + r = pqr - 2$.

6. Express $x^2 - 6x - 10$ in the form $(x+a)^2 + b$.

7. Given that α and β are the roots of the equation $5x^2 - 3x + 2 = 0$, find the equation whose roots are $\frac{2}{\alpha - 2}$ and $\frac{2}{\beta - 2}$.

8. Determine the distance between the points of intersection of the curve $5x^2 + 6xy - 8y^2 = 0$ and the line $3x - y = 7$.

9. Find the value of x for which $\log_2 x - \log_x 4 = 1$.

10. Solve the simultaneous equations.

    (i) $2a - 3b + c = 10$
    $a + 4b + 2c + 3 = 0$
    $5a - 2b - c = 7$

    (ii) $3x + y + z = 0$
    $x - 2y + z = 4$
    $2x - y - z = 5$

    Hint: In (i) and (ii), by eliminating *one of the letters*; taking any two equations at a time, we can solve the equations.

11. Express $\frac{1+\sqrt{3}}{(\sqrt{3}-1)^3}$ in the form $a + b\sqrt{c}$.

12. Find the maximum value of $1 - 5x - 2x^2$ and the value of x for which it occurs.

13. Solve the equations (i) $2\sqrt{x} - \sqrt{x+5} = 1$ (ii) $\sqrt{p+6} - \sqrt{p+3} = \sqrt{2p+5}$.

14. If $y = a + bx^n$ is satisfied by the values in the table below,

x	1	2	4
y	7	10	15

Show that $n = \log_2 \frac{5}{3}$ and deduce the values of *a* and *b*.(Hint: Form 3 equations and solve them simultaneously)

15. Solve the equations:

(i) $\log(x-1) + 2\log y = 2\log 3$, $\log x + \log y = \log 6$

(ii) $\log_2 x^2 + \log_2 y^3 = 1$, $\log_2 x - \log_2 y^2 = 4$

Hint: Apply laws of logs

16. Solve the equations

(i) $\log_{2x} 4 = \log_4 x$ (ii) $(\log_3 x)\cdot(\log_{3x} 3) = \frac{3}{4}$

(iii) $2\sqrt{x} + \sqrt{2x+1} = 7$ (iv) $t^2 + 2t = 34 + 35/(t^2 + 2t)$

17. The points O, R and S have coordinates $(0\ ,\ 0)$, $(4,\ 3)$ and $(a\ ,\ b)$ respectively. Given that OR and RS are perpendicular,

(i) show that $4a + 3b = 25$, (ii) find the value of *a* if $b = 4$.

18. Line L_1 is defined by the equation $2x + y = 1$. L_2 is a line through point $(2,\ -1)$ and is perpendicular to line L_1. Calculate the perpendicular distance of the point $(0\ ,\ -4)$ from L_2.

19. A triangle ABC has sides with equations $3x - y = 3$, $x - y - 1 = 0$ and $3x - 5y = -9$ respectively. Find the area of the triangle ABC.

20. The points A and B have coordinates $(2\ ,\ 4)$ and $(1\ ,-3)$ respectively. Determine the coordinates of the point T which divides AB internally in the ratio $2:1$.

21. Calculate the acute angle between the straight lines $3x - y = 5$ and $x + 4y = 11$.(Use two methods)